

FRAMEWORK FOR PHILIPPINE MATHEMATICS TEACHER EDUCATION

Department of Science and Technology
SCIENCE EDUCATION INSTITUTE

Philippine Council of Mathematics Teacher Education (MATHTED), Inc.

Framework for Philippine Mathematics Teacher Education

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Citation:

SEI-DOST & MATHTED, (2011). *Framework for philippine mathematics teacher education*. Manila: SEI-DOST & MATHTED.

ISBN 978-971-8600-47-4

Published by:

Science Education Institute, Department of Science and Technology
1st and 2nd Levels, Science Heritage Building
DOST Compound, General Santos Avenue
Bicutan, Taguig City, Metro Manila, Philippines
Tel. Nos. (632) 837-1359, (632) 839-0241, Fax No. (632) 837-1924
<http://www.sei.dost.gov.ph> / www.science-scholarships.ph and

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Printed in Metro Manila, Philippines

Foreword

This framework is the product of months of careful planning and discussions, with ideas coming from the best minds in the field of mathematics, prior to the actual drafting of the manuscript. Although there may have been opposing views during the development of this framework, which is not unusual when experts meet, the final output is proof that individuals with diverse backgrounds and beliefs could be united by a common vision and goal.

The *“Framework for Philippine Mathematics Teacher Education”* contains resources that could guide higher education institutions, professional organizations of mathematics teachers and school administrators in assessing and improving the performance and career development of mathematics teachers based on a set of standards. The qualities of effective mathematics teachers in terms of what they should know (content knowledge), what they are expected to do to achieve quality learning outcomes (pedagogical knowledge) and what they should possess to be able to manage the different aspects of the teaching and learning process (management skills) upon which the indicators are based are also included. All these are anchored on the objective of raising the quality of mathematics education.

It is hoped that this framework will be widely used and applied by the various stakeholders, and that together we will work towards achieving the desired goal of effective mathematics teaching among our future teachers.

Dr. Filma G. Brawner

Director, Science Education Institute

PREFACE

Mathematics teacher education in the Philippines is in its most precarious state. The ongoing MIEA International Study on Teacher Education Development, Mathematics Component will surely lead us to question our practices in developing future teachers of mathematics. This framework document puts together the most essential ideas that highlight the most important domains of knowledge that mathematics teachers should develop as they prepare for their future job in schools.

Excellence in teaching mathematics is the desired goal for our future teachers. This document is intended for university professors of mathematics education and educational leaders who care about how our teachers grow and develop as one of the most important citizens of our country in the next decades. With this framework, we give credence to our teachers who continue to make supreme sacrifices despite the many obstacles that they face. We believe in their ability to overcome substandard preparatory programs, inadequate teaching facilities and materials, low wages and poor regard from their fellow countrymen. But, we all have to help. With this framework, teacher education institutions can no longer offer excuses not to educate our future mathematics teachers in the best ways possible.

The Philippine Council of Mathematics Teachers Educators (MATHTED), Inc. and the Science Education Institute of the Department of Science and Technology present this Framework for Philippine Mathematics Teacher Education. We hope that this document will open more eyes, minds and hearts in order that we may all work together for a brighter future for our teachers and consequently, a secure life for our children.

Catherine P. Vistro-Yu, Ed.D.
Project Director and Lead Researcher
Mathematics Framework Project
(2005 – 2008)

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CHAPTER 1

INTRODUCTION

Achieving excellence in mathematics teaching is the first step towards professionalizing mathematics teachers. To do this, mathematics teachers should aim to pursue a high quality education. Through its course offerings, practicum or field experience and support structures, teacher education institutions (TEIs), for their part, must offer excellent education to future mathematics teachers and not settle for anything less.

The many challenges that mathematics teachers and educators face today make mathematics teaching especially difficult. Foremost among these challenges is the amount and depth of mathematics content that teachers ought to master. For example, an elementary mathematics teacher's belief that any number divided by zero is also zero shows that a deep understanding of mathematics content is sorely lacking among our mathematics teachers. Directly linked to this is our mathematics teachers' poor preparation in identifying effective pedagogies for teaching specific content material to particular groups of learners. It is not uncommon to observe poorly trained mathematics teachers who either teach very low levels of skills to more able students or teach advanced skills to students who lack the prerequisite knowledge or skills. Mathematics teachers also find it daunting to implement some general learning strategies such as the use of cooperative learning and also to manage their students that are engaged in such learning activities. On top of these

expectations, mathematics teachers need to display encouraging behaviors and attitudes as well as engage themselves in a life-long professional development program.

In sum, we find that mathematics teachers are tested by:

1. the amount and depth of content in mathematics that is available for them to learn so that they could teach good and correct mathematics to students;
2. the varied cognitive backgrounds of students requiring a wide range of pedagogical approaches to learning mathematics;
3. the unpredictability of students' contexts and behavior these days that require teachers to be armed with multiple ideas for managing students, class behavior and resources;
4. the existence of various types of technologies and their rapid advancements;
5. the perceived disconnect between school mathematics and everyday life;



6. their role as models of positive values and attitudes, which would carry students far in their lives and careers, and;
7. the need to continuously develop themselves in the teaching profession.

Such challenges hamper mathematics teachers' creative abilities and self-efficacy. Mathematics teachers clearly need all the extra help that they can get to achieve excellence in the teaching of mathematics at the school level.

This framework provides a set of standards that could guide TEIs, state universities and colleges (SUCs), professional organizations of mathematics teachers, schools and other educational groups involved in the educational and professional development of mathematics teachers of grade levels K-10/11 in the Philippines. It offers a vision of what a competent mathematics teacher is and identifies the knowledge components that mathematics teachers should possess. This document maps out a professional development continuum for mathematics teachers and outlines performance expectations at each growth level. With the numerous demands from mathematics teachers, such a framework is needed to ensure high quality standards in mathematics education.

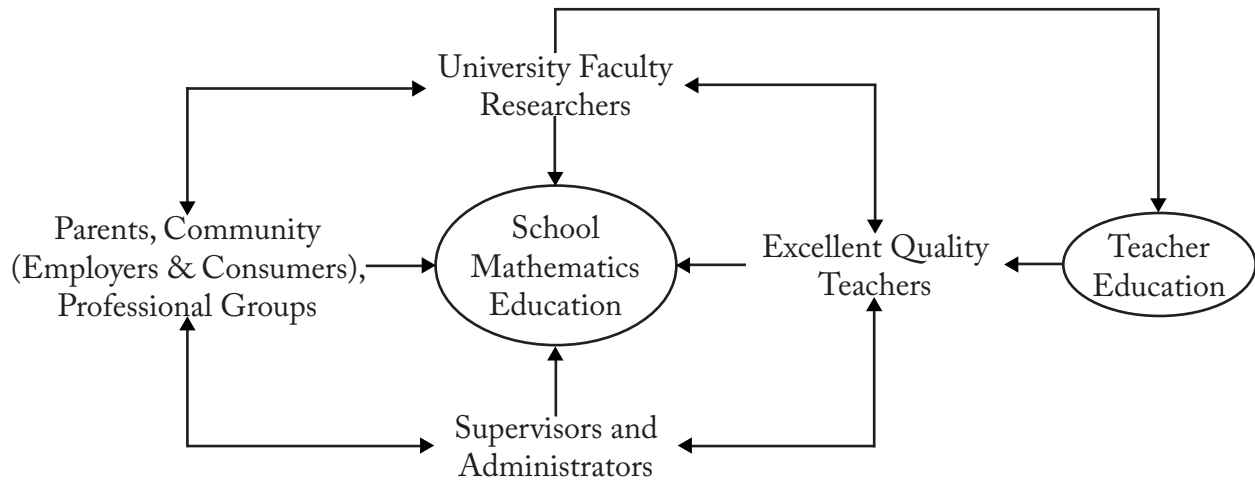
This document is an expression of a principled stand on how mathematics teachers should be educated and what they deserve to gain from their preparatory and continuing education. Therefore, any person, group or institution that wishes to engage in the education and professionalization of mathematics teachers are urged to be mindful of this framework and to adhere to the principles and standards espoused by this document.

Scope of the Framework

There are larger problems that remain as almost permanent challenges to mathematics teachers. These include large class sizes in many elementary and secondary schools, lack of good quality textbooks, lack of physical space for learning (i.e., classrooms, laboratories, etc.) and lack of qualified personnel including teachers. These problems are what Nebres (1983) refer to as macro problems – problems that require massive efforts by government to solve due to their scope and pervasiveness. While we submit that these macro problems are important to address, this framework does not address such problems. These problems are beyond the scope and intent of this document. However, we are concerned with them and, therefore, accept the challenge of helping address these macro problems in a future document or forum.



Figure 1. A Model for Improving Mathematics Education



The ultimate goal is to raise the quality of school mathematics education to world standards. To do so requires the close coordination among mathematics teachers who are likewise assumed to have met world-class standards, school supervisors and administrators, university faculty and researchers and the community at large. Each of these groups contributes in many ways to the improvement or decline in school mathematics education. At the same time, university faculty and researchers also contribute to the quality of education at both the pre-service and in-service levels for mathematics teachers.

– *Framework for Mathematics Teacher Education, A Working Draft, 2006*

CHAPTER 2

DECLARATIONS

Any document that seeks to influence decision makers and empower the lesser individuals in the educational arena must be grounded in principles that are shared by all people concerned. The following nine (9) non-negotiable principles guided the development of this framework. One must keep these in mind when reading and understanding the components of the framework. These are principles that must prevail in our pursuit of good quality education for developing highly competent mathematics teachers.

Principle 1. While the ability to explain and solve a problem is evidence of good understanding of some mathematical ideas, teaching mathematics requires much more than these.

Teaching mathematics does not only mean knowing how to explain or to show how a problem is worked out. Teaching mathematics requires a deep understanding of principles and theories behind every single mathematics problem that is solved. The saying that “one cannot give what one does not have” is very true. Mathematics teachers ought to possess strong knowledge of mathematics content. They must not be satisfied with knowing just enough mathematics but rather aim for a deeper level of mathematical understanding. They must not be satisfied with knowing just one or two ways of solving problems but rather try to understand every possible solution or approach to a problem. By having a deep understanding and knowledge

of mathematics, teachers are better able to guide students into learning mathematics well.

However, more than knowing their mathematics, teachers ought to know how to lay out school mathematics content and break them down into manageable chunks of material to learn. In order to do this, mathematics teachers must know the why and how of mathematics in addition to the what of it. How teachers learn their mathematics is also important because that is the experience they bring when they teach (Ball and Bass, 2004).

Principle 2. Mathematics must be real to students and therefore, mathematics teachers should be mindful of students’ contexts when teaching mathematics.

One value of mathematics in a student’s education is its usefulness in life. Students must therefore be able to connect mathematics



to their everyday lives. Mathematics teachers should consider students' backgrounds and contexts when teaching mathematics in order for students to understand the importance and uses of mathematics in real life. For example, it does not make sense to teach mathematics using technology in schools when both schools and students have limited resources and therefore, do not have the technology needed. Rather, mathematics teachers must be resourceful and think of alternative approaches to teaching mathematics that does not require the use of technology. Another example is when mathematics teachers use applications to real life situations that are very different from students' personal experiences. Teachers must use situations that students are familiar with and not some experience that alienates them from mathematics. The landmark study of Carraher, Carraher and Schliemann (1985) on the Brazilian street children certainly brought home this point.

On the other hand, the relevance of the mathematics being taught does not always have to be immediately evident. Certain foundational mathematical principles have no apparent connection to real life situations but are necessary to teach. In these situations, it is important that mathematics teachers know how to, at the very least, speak the "language" of the students, which includes knowing the language they speak, the jargon they possibly use as well, and all the accompanying nuances of their language so that mathematics is still understandable to the students.

Principle 3. Mathematics is best learned when students are actively engaged.

Mathematics is not a spectator sport. Students must be engaged in the learning activities planned by the teacher for them to learn mathematics (Bernardo, 1998). Therefore, students cannot expect to learn by simply watching their teacher

solve problems on the board. In fact, students must bear the responsibility of being actively engaged in order to maximize their learning potential. They ought to join in discussions, ask questions, argue and reason out so that they see the many different aspects of mathematics that they are studying. Likewise, even while their mathematics teacher works out sample problems in class, students, too, must do the problem themselves because doing so helps them learn and remember the skills and processes used.

Mathematics students can still be engaged even while listening to the teacher. The key is for the teacher to be able to catch and sustain students' attention throughout the class period while at the same time make them realize their responsibility as learners. Mathematics teachers must, therefore, remember to carefully select learning activities that are interesting and stimulating. They should give students adequate time to reflect on their own thinking and make connections between their prior knowledge and the new knowledge. They should also remember to include students in class discussions, encourage them to ask questions, allow them to argue and make conjectures in order to develop students' reasoning skills.

Principle 4. Mathematics can never be learned in an instant, but rather requires lots of work and the right attitude.

The mathematics that is being learned in schools today is a product of centuries-old discoveries, inventions and experimentations by mathematicians that have been achieved through many, many years of hard work, both individual and collaborative. Mathematics is not an easy subject to learn. It cannot be learned in an instant. Just as our predecessors have invested a lot of their time, talent and resources so must today's learners of mathematics also invest time to learning and understanding it. We believe that students must possess the right attitudes



to be able to learn mathematics. They must have patience and a good attitude towards work in order to learn mathematics. Mathematics teachers, themselves, must understand that most students take time to learn (Putnam, Heaton, Prawat and Remillard, 1992). Thus, they must be patient with students and not expect instant results from them.

Principle 5. All students regardless of sex, culture, socio-economic status, religion and educational backgrounds have the right to learn and be taught good and correct mathematics.

Mathematics is not just for an elite group of students. While certain groups of learners (e.g., males, the Chinese) have been recognized because of their seemingly “natural talent” in mathematics, mathematics is not meant just for them. All students regardless of sex, culture, socio-economic status, religion and educational backgrounds deserve to learn and be taught good and correct mathematics (Gates and Vistro-Yu, 2003). Mathematics has been proven to improve the quality of life through its many applications in many aspects of the human life. Everyone should receive a high quality mathematics education. Oftentimes, mathematics teachers expect less from students who come from minority groups or underprivileged environments. This practice must stop because all students deserve to be treated equally no matter what background they come from. This means that students are expected to work as hard and study mathematics as seriously as all other students in schools around the country.

Principle 6. Assessment must be an integral part of mathematics instruction.

Mathematics teachers must realize the importance of the role of assessment in improving the teaching-learning process. It is

crucial that teachers recognize the alignment of curriculum, instruction and assessment. Thus, it is necessary to ensure that the goals of the curriculum are achieved through appropriate instructional strategies and assessment. This can be done if teachers constantly reflect on their teaching, that is, analyze whether what was intended to be taught has actually been taught and learned by the students. Moreover, instruction can be diagnostic in nature. It can be designed to help identify misconceptions. It can provide learning experiences that allow students to attain conceptual change and maximize learning. Consequently, assessment is integrated into the teaching process.

Principle 7. Mathematics as a field continues to develop and evolve. Therefore, the teaching of it must keep up with developments in the field.

Mathematics teachers must realize that the field of mathematics continues to grow. More theories are developed while new processes and solutions continue to evolve or be discovered. The teaching of mathematics must be dynamic, constantly keeping up with new trends and developments in the field. Mathematics teachers must keep in mind the practical value of mathematics – its usefulness in a constantly changing world. Therefore, they must be open to developing new techniques in teaching to parallel the developments in the field and in the world.

Principle 8. Technology plays an important role in the teaching and learning of mathematics. Mathematics teachers must learn to use and manage technological tools and resources well.

No one can deny that technology plays a huge role in the learning of mathematics. With more and more powerful computers invented and developed everyday, the learning of mathematics



has become a lot easier for many students (Heid, 2005). With software that allows for easy manipulation of variables and provides picture perfect representations of mathematical components and entities, mathematics has become much more accessible and real.

Technology has also made the teaching of mathematics more exciting and less complicated. With calculators and computers, mathematics teachers are now able to plan real problem solving and modeling activities for students to learn real and useful mathematics concepts. On the other hand, because these tools can easily perform mathematical tasks, students tend to rely too much on them, to the point of being too lazy to think and perform even the simplest of calculations. Mathematics teachers must teach students to use technology judiciously, keeping in mind that these tools are meant simply to assist students in their learning of mathematics.

Principle 9. Mathematics teachers must never stop learning.

It is the responsibility of mathematics teachers to continue learning new ideas both about mathematics and about the teaching of mathematics. They must continue to grow both as teachers and learners of mathematics. As facilitators of learning, mathematics teachers must ensure their own personal and professional growth by engaging in activities that allow them to learn new methods and ideas and produce learning support materials that will help in the teaching of mathematics.

Mathematics teachers must learn to engage in collaborative work that contributes to their further development. They must also learn how to optimize its use with colleagues and maintain an atmosphere that encourages sharing of ideas and professional support. Mathematics teachers must also bear the responsibility of reaching out to the community to which they belong, helping other people to learn, appreciate and use mathematics.



Teaching Principles

- Principle 1. While the ability to explain and solve a problem is evidence of good understanding of mathematical ideas, teaching mathematics requires much more than these.
- Principle 2. Mathematics must be real to students and therefore, mathematics teachers should be mindful of students' contexts when teaching mathematics.
- Principle 3. Mathematics is best learned when students are actively engaged.
- Principle 4. Mathematics can never be learned in an instant, but rather requires lots of work and the right attitude.
- Principle 5. All students regardless of sex, culture, socio-economic status, religion and educational backgrounds have the right to learn and be taught good and correct mathematics.
- Principle 6. Assessment must be an integral part of mathematics instruction.
- Principle 7. Mathematics as a field continues to develop and evolve. Therefore, the teaching of it must keep up with developments in the field.
- Principle 8. Technology plays a key role in the teaching and learning of mathematics. Mathematics teachers must learn to use and manage technological tools and resources well.
- Principle 9. Mathematics teachers must never stop learning.

Learning Principles

- Principle 1. Being mathematically competent means more than having the ability to compute and perform algorithms and mathematical procedures.
- Principle 2. The physical and social dimensions of a mathematical environment contribute to one's success in learning mathematics.
- Principle 3. Mathematics is best learned when students are actively engaged.
- Principle 4. A deep understanding of mathematics requires a variety of tools for learning.
- Principle 5. Assessment in mathematics must be valued for the sake of knowing what and how students learn or fail to learn mathematics.
- Principle 6. Students' attitudes and beliefs about mathematics affect their learning.
- Principle 7. Mathematics learning needs the support of both parents and other community groups.



CHAPTER 3

A VISION

Excellence in mathematics teaching can be achieved with the following vision:

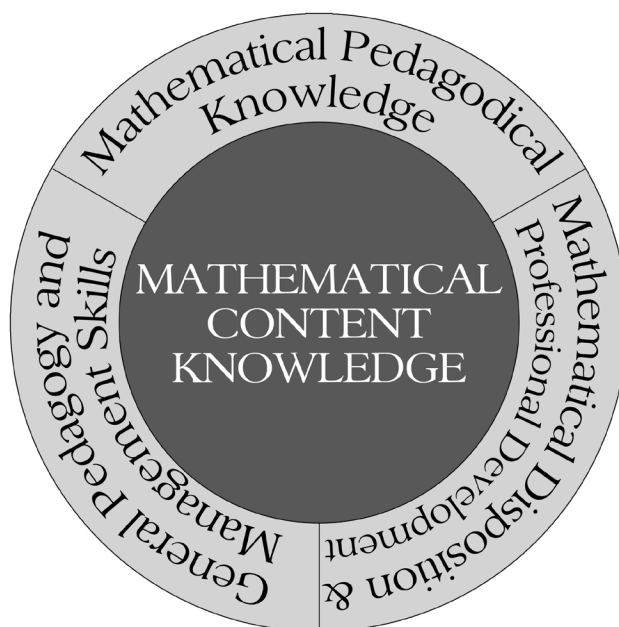


Figure 2. Knowledge and Skill Components of Mathematics Teaching

Vision of a Fully Competent Mathematics Teacher

A fully competent mathematics teacher possesses a strong mathematical content knowledge, is armed with mathematical pedagogical knowledge as well as general pedagogical knowledge and management skills, displays an appropriate mathematical disposition and values one's own professional development.

Mathematics Content Knowledge

This refers to mathematics teachers' knowledge of, understanding of and competencies in the contents of mathematics.

Mathematical Pedagogical Knowledge

This refers to mathematics teachers' understanding and use of teaching approaches, learning theories, assessment principles and modalities particular to mathematics.

General Pedagogical Knowledge

This refers to mathematics teachers' understanding and use of teaching approaches, learning theories and modalities in general contexts and environments.

Classroom Management Skills

This refers to mathematics teachers' approaches in providing for a favorable environment for student learning which includes organizing



the physical set-up, providing alternative modalities and modes for learning and the use of equipment, tools, kits, gadgets and facilities needed to effectively teach mathematics.

Mathematical Disposition

This refers to mathematics teachers' ability to make decisions for themselves that would impact on students' learning and on their own professional development. It includes mathematics teachers' beliefs and attitudes, their inclination to use mathematics and their willingness to reach out to others.

Professional Development

This refers to mathematics teachers' level of advancement in the teaching profession beyond the minimum required degree and licensure for employment, as well as their involvement in research and production of learning support materials.

Levels of Teachers' Growth and Development

To assist mathematics teachers in their pursuit of excellence and perfection in their profession, the framework provides a continuum of development, mapping out the levels of growth that teachers could achieve in their profession. There are four levels and each is described below:

Novice

This refers to a mathematics teacher who had just finished the required tertiary education degree and has passed the Licensure Examination for Teachers (LET) and other requirements. This also refers to an individual who has a different profession but had just shifted careers to teach mathematics. A novice teacher is also someone who may have a tertiary education degree different from the required degree but should have fulfilled the requirements equivalent to the required tertiary degree and has passed the LET or is at least on the way to acquiring the required license to teach. His or her content knowledge

may be weak. Beyond this, the novice teacher sticks to a certain formula in teaching and therefore, lacks depth and is quite inflexible. The person is basically inexperienced in many aspects of teaching and handling a mathematics class.

Emerging

This refers to a mathematics teacher who possesses better qualities than the novice teacher and shows promise in the mathematics teaching profession. The teacher's strong mathematical foundation is apparent in the kinds of materials used in teaching. The teacher more or less knows which pedagogy and teaching strategies are effective for certain groups of students and practices these. On the other hand, the emerging teacher is still learning some skills in managing students, classrooms and other resources in the class. The emerging mathematics teacher is not yet consistent in making the right choices in relation to students and mathematics learning, teaching and professional development.

Accomplished

At this level, a teacher possesses better qualities than the emerging teacher and the content knowledge is much more solid. An accomplished mathematics teacher knows the effective pedagogies and strategies for mathematics and is able to manage the classroom learning environment satisfactorily. The accomplished teacher is more flexible than the novice or emerging teacher. This teacher can adjust to different classroom situations. At this level, the mathematics teacher's disposition in mathematics and quest for new knowledge are worth emulating.

Expert

At this level a mathematics teacher is beyond "accomplished". The expert teacher is successful in all components – possesses very strong content knowledge, has deep understanding of content, mathematics pedagogy and general pedagogy



so that the teacher could easily adjust according to the situation. The expert teacher excellently blends theory and practice in mathematics education. The expert also displays a genuine desire to champion the cause of mathematics education. Thus, a mathematics teacher who is labeled an expert possesses the characteristics of the “accomplished” and is willing to advocate for and work towards the continued development of mathematics teaching and learning.

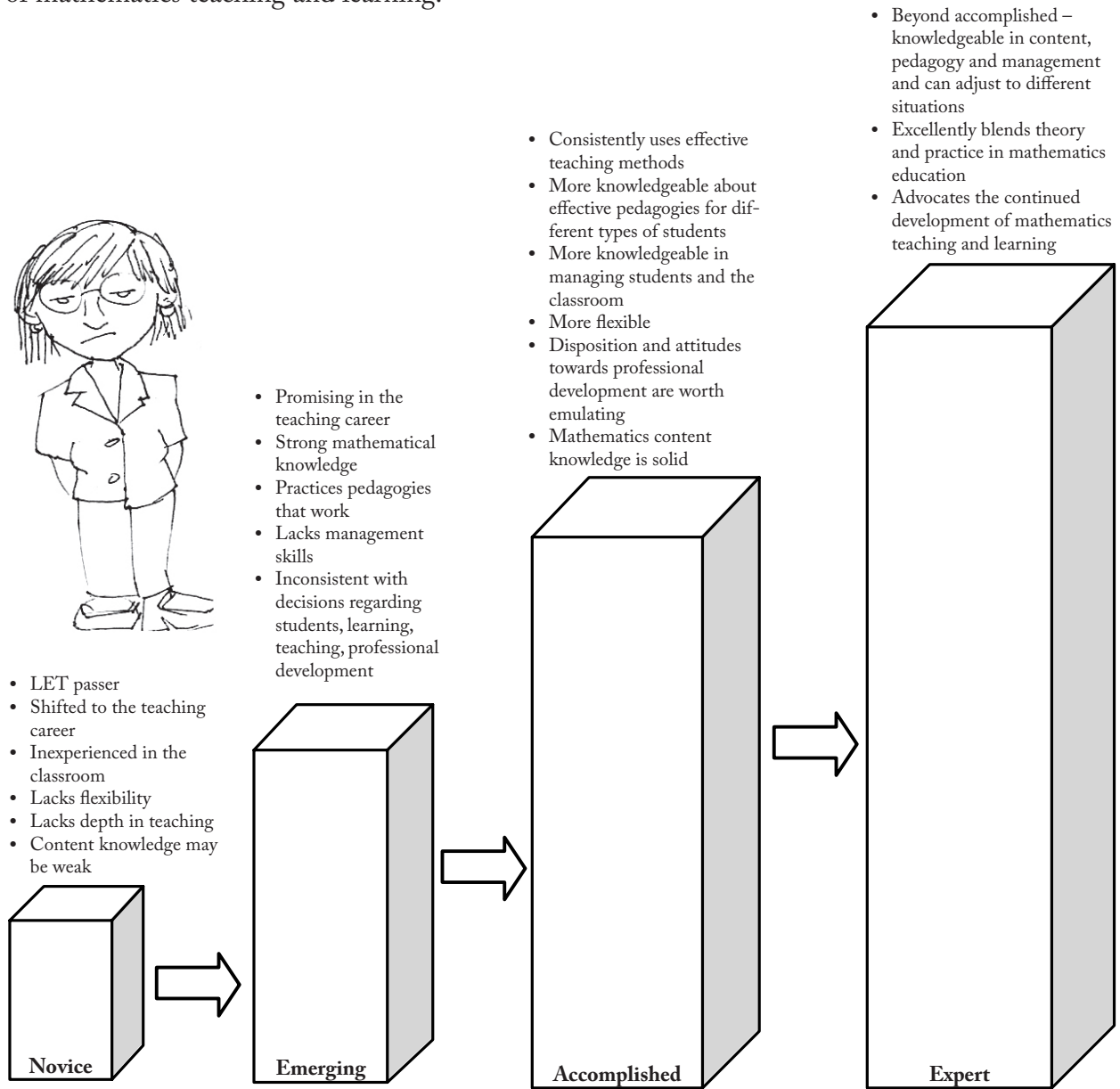


Figure 3. Highlighted Components Describing Levels of Teachers’ Growth and Development

Sketch by Martin Kristopher R. Limgenco



CHAPTER 4

MATHEMATICS CONTENT KNOWLEDGE OF TEACHERS

At the heart of all competencies of mathematics teachers is their competency in mathematics. The primary responsibility of mathematics teachers to their students is to know and understand very well the mathematics content that they are assigned to teach, and more. This is a fundamental requirement. For how could they teach mathematics effectively to their students if they themselves do not understand it? This is the first part of Principle 1 of this framework.

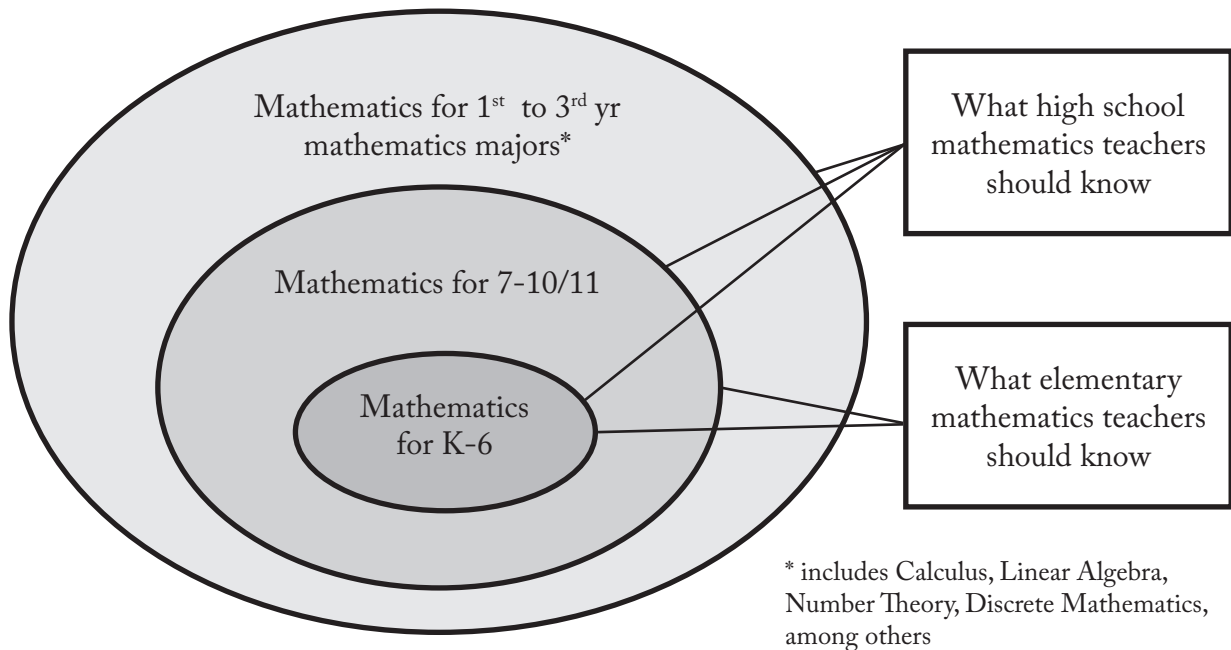


Figure 4. Recommended Content Expertise and Exposure of K-10/11 Mathematics Teachers in Comparison to Mathematics in Schools and Universities.

Figure 4 shows the mathematics that teachers at the K-10/11 levels should know or have been exposed to in comparison to what students at these levels are supposed to learn. Mathematics teachers at the K-6 levels should know the K-9/10 mathematics taught in elementary and secondary schools and must have been exposed to mathematics at level 10/11. Mathematics teachers at the 7-10/11 levels should know the mathematics taught in elementary and secondary schools plus some more major mathematics courses taught at the first two or three years of college level courses taken by mathematics majors in schools of sciences (i.e., BS Mathematics) of universities and colleges.

Mathematics teachers at the K-6 grade levels should display the right amount of knowledge and competencies in the following areas:

- Numbers and number sense (Number Theory);
- Patterns, symbols, functions (Algebra and Trigonometry);
- Properties of geometric figures and measurements (Metric, Non-metric, Trigonometry, and Analytic Geometry);
- Sets;
- Contemporary mathematics (Discrete Mathematics);
- Data handling, counting techniques, measures of chance (Probability and Statistics).

On the other hand, mathematics teachers at the 7-10/11 grade levels should display mastery in the aforementioned areas as well as in the areas of:

- Calculus;
- Additional algebras such as Linear Algebra and Abstract Algebra;
- Set theory;
- College level Number Theory;
- Logic.

Content Emphases

Table 1 on the next pages indicates the content knowledge emphases in the content preparation of elementary and secondary school mathematics teachers. The content emphases have been classified according to the content classification used in the Trends in International Mathematics and Science Study (TIMSS) in 2003, called content strands.

Philippine Teacher Education

Principle 1

While the ability to explain and solve a problem is evidence of good understanding of some mathematical ideas, teaching mathematics requires much more than these.



Table 1. Matrix of content emphases in the K-10/11 mathematics curriculum that mathematics teachers should know

Content Strands	Mathematics Teachers of K-6	Mathematics Teachers of 7-10/11
	The teachers of K-6 should show content competencies by	The teachers of 7-10/11 should demonstrate the competencies expected of teachers of K-6, as well as the following:
Numbers and Number Sense	<ul style="list-style-type: none"> • Describing correctly the structure and properties of complex numbers: real numbers (counting, whole, integers, fractions, decimals, percent, ratio & proportion, rational, irrational numbers) and non-real numbers; • Solving problems involving these numbers; • Posing problems involving these numbers; • Making correct conjectures based on observed numerical patterns and relationships, and verifying results; • Proving fundamental theorems involving numbers. 	<ul style="list-style-type: none"> • Describing correctly the structure and properties of other number systems (e.g., congruence-modulo, and the like); • Making conjectures involving numbers in these systems; • Proving conjectures and theorems involving numbers in these systems.
Measurement	<ul style="list-style-type: none"> • Defining the different terms in measurement and describing the attributes of each: length, mass, weight, time, temperature; perimeter, circumference, area, surface area, volume, angle measurement, scales, rate, speed, velocity; • Deriving the formula for perimeter, area, volume, and surface area of various shapes and solids; • Describing and comparing mathematical and real world objects using well selected and appropriate units and tools; • Solving problems involving these measurement ideas; • Posing problems involving these measurement ideas; • Making conjectures on measurement. 	<ul style="list-style-type: none"> • Proving conjectures and theorems involving measurement; • Defining the different terms in measurement and describing the attributes of each: density, successive approximations, upper and lower bounds, limits; • Deriving the formula for area of plane figures and volume of solids using integrals; • Solving problems on area and volume using integrals.



Content Strands	Mathematics Teachers of K-6	Mathematics Teachers of 7-10/11
	The teachers of K-6 should show content competencies by	The teachers of 7-10/11 should demonstrate the competencies expected of teachers of K-6, as well as the following:
Geometry	<ul style="list-style-type: none"> • Describing properties and relationships of basic concepts in the axiomatic Euclidean geometry (points, lines, planes and angles); • Using appropriate objects or manipulative materials to represent geometric terms; • Demonstrating geometric connections; • Describing properties of plane and solid figures; • Free-hand drawing of geometric figures based on a given description; • Constructing geometric figures based on a given description using a compass; • Classifying attributes of different kinds of shapes; • Demonstrating the use of algebra to verify the properties of plane and solid figures; • Solving and posing problems involving geometric figures; • Proving theorems involving geometric concepts using inductive and deductive reasoning; • Making conjectures about properties of shapes including transformations and combinations of shapes, and verifying these conjectures. 	<ul style="list-style-type: none"> • Describing properties and relationships of basic ideas of non-Euclidean geometry; • Using mathematical investigations to explore connections among geometric ideas and real world situations; • Demonstrating the use of analytic geometry to verify and prove properties of geometric figures.
Patterns, Functions, and Algebra	<ul style="list-style-type: none"> • Solving equations and inequalities; • Solving mathematical problems based on real world situations, which include non-routine problems; 	<ul style="list-style-type: none"> • Solving mathematical problems using: <ul style="list-style-type: none"> • Matrices and vectors • Trigonometry • Limits and derivatives; • Recognizing properties of basic algebraic structures (e.g., groups).



Content Strands	Mathematics Teachers of K-6	Mathematics Teachers of 7-10/11
	The teachers of K-6 should show content competencies by	The teachers of 7-10/11 should demonstrate the competencies expected of teachers of K-6, as well as the following:
Patterns, Functions, and Algebra <i>(continuation)</i>	<ul style="list-style-type: none"> • Posing algebraic problems based on real world situations; • Using different representations and models of a given real world situation; • Recognizing patterns and making conjectures based on these observed patterns using functions; • Proving properties of equations and inequalities; • Working with all types of functions (e.g., algebraic and non-algebraic). 	
Data, Analysis and Probability	<ul style="list-style-type: none"> • Demonstrating skills of collecting, organizing, reading, representing and interpreting data; • Describing terms in counting techniques and probability; • Solving problems involving the measures of central tendencies, and measures of dispersions; • Making predictions about outcomes and verifying these predictions using intuitive approaches. 	<ul style="list-style-type: none"> • Differentiating inferential from descriptive statistics; • Solving problems involving <ul style="list-style-type: none"> • inferential statistics • probability • probability distributions • and mathematical expectations; • Demonstrating skills in analyzing and interpreting data; • Making predictions about outcomes and verifying these predictions through inferences.

Content Knowledge Domains

We organize content knowledge according to four domains. The second part of Principle 1 basically reflects the need for teachers to be competent in each of these four domains:

- Mathematical Concepts;
- Mathematical Processes;
- Communication;
- Connections.

The domain of mathematical concepts refers to the content topics, principles and theories within the field and subfields of mathematics.

The domain of processes refers to the skills and processes in mathematics. These include computing, problem solving, reasoning and proving. The domain of communication refers to the language of mathematics, its notations, symbols and various representations. The domain of connections refers to the applications of mathematics to real life situations as well as to other disciplines.



Performance Indicators

Table 2 contains a list of performance indicators expected of mathematics teachers within the mathematics content knowledge domains. This table lists the kind of knowledge and skills expected of mathematics teachers according to the knowledge domains Mathematical Concepts, Mathematical Processes, Communication and Connection and the behavior that indicates success in acquiring the desired knowledge. The table also specifies the standards at which each performance indicator is expected to be manifested by mathematics teachers according

to whether they are novice, emerging, accomplished or expert teachers. Ultimately, it is these standards that will indicate the growth level that a mathematics teacher is in.

The stars denote how well mathematics teachers are able to manifest the performance indicator based on the quality of the performance, the consistency of the action or behavior, and the intensity or passion that teachers bring into their performance.

*Table 2. Performance indicators by domain of mathematical content knowledge.
N - Novice; Em - Emerging; A - Accomplished; Ex - Expert*

† - Barely meets the expectation

* - Greatly meets the expectation

★ - Substantially meets the expectation

✱ - Exceeds expectation

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Mathematical Concepts	<i>The teacher knows and understands the concepts and principles of mathematics and its sub-areas or sub-disciplines.</i>				
	The teacher passes a well designed content examination for mathematics teachers (i.e., qualifying exam, not necessarily the LET).	★	★	★	★
	The teacher receives a passing rate in the performance evaluation in terms of knowledge of content.	†	★	★	✱
	<i>The teacher is aware of the assumptions and prerequisites of each mathematical concept.</i>				
	Reflected in the teacher's lesson plans are appropriate pre-requisite skills and knowledge of the current lessons.	†	★	★	✱
	The teacher can arrange mathematics topics following their logical sequential order.	†	★	★	★
Mathematical Processes	<i>The teacher understands the basic thinking processes.</i>				
	The teacher demonstrates facility in using basic thinking skills, e.g., identifying, classifying, verifying, comparing and contrasting.	†	★	✱	✱



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Mathematical Processes (continuation)	<i>The teacher understands the higher order thinking processes, e.g., problem solving, reasoning and proving.</i>				
	The teacher can solve problems, analyze errors, reason out, formulate decisions, make conjectures and prove theorems or conjectures.	★	★	★	★
	The teacher demonstrates varied ways of:				
	solving a problem;	★	★	★	✱
	making conjectures;		★	★	★
	reasoning;	★	★	★	✱
	proving theorems or conjectures;		★	★	★
	making decisions	★	★	★	✱
	The teacher can generalize results of problems and extends them to other problem situations.	★	★	★	✱
	The teacher is able to apply mathematical reasoning and appropriate technologies in the development of concepts, procedures and conjectures.		★	★	★
	The teacher generalizes reasoning skills within the study of mathematics and applies or extends them to other contexts.		★	★	★
	The teacher is able to use problem explorations and modeling to extend the mathematical understanding of students.		★	★	★
Communication	<i>The teacher understands the language of mathematics and the use of representations in mathematics.</i>				
	The teacher demonstrates competence in the use of the language of mathematics – its specialized vocabulary, symbols and graphs to represent and describe mathematical ideas, generalization and relationships.	★	★	✱	✱
	The teacher uses precise and developmentally appropriate language of mathematics.	★	★	✱	✱
Connection	<i>The teacher uses real world situations to build mathematical knowledge.</i>				
	The teacher employs objects and situations within the student’s context in developing conceptual understanding.	★	★	✱	✱
	<i>The teacher can apply mathematical content to real world situations.</i>				
	The teacher includes activities in the lesson plan activities that show connection of the concepts to real world situations.	★	★	★	★



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Connections (<i>continuation</i>)	<i>The teacher understands the connections within the mathematics curriculum.</i>				
	(<i>continuation</i>)	✦	★	★	✦
	<i>The teacher understands mathematical connections to school curricula and to other disciplines.</i>				
	The teacher connects mathematics to other disciplines.	✦	★	★	✦

Quest for Excellence in Mathematics Content Knowledge

It is helpful to pull together the ideas offered in Table 2 so that mathematics teachers and educators know how to proceed to achieve excellence in the mathematics content knowledge. The model for excellence is the Expert mathematics teacher. However, it is true that not many teachers reach this level of growth and performance in their career. The more realistic objective is for all mathematics teachers to aim to become an Accomplished teacher. In this section, the expectations of Expert and Accomplished mathematics teachers with respect to mathematics content knowledge are summarized. Based on Table 2, Expert mathematics teachers exceed expectations in the following areas:

- Passing performance evaluation;
- Writing of lesson plans that reflect prerequisite knowledge and skills;
- Use of basic thinking skills;
- Use of varied ways of solving a problem, reasoning and making decisions;
- Generalizing and extending results of solving problems;
- Use of the language of mathematics;
- Use of appropriate objects or situations to develop students' mathematical understanding;
- Constructing connections within and among the branches of mathematics;
- Connecting mathematics to other disciplines.

Expert mathematics teachers greatly meet the expectations indicated by the following behaviors:

- Arranging mathematical topics in their logical sequential order;
- Solving problems, analyzing errors, reasoning out, formulating decisions, making conjectures and proving theorems or conjectures;
- Using varied ways of making conjectures and proving theorems or conjectures;
- Applying mathematical reasoning and appropriate technologies to develop concepts, procedures and conjectures;
- Generalizing reasoning skills and applying or extending them to other contexts;
- Using problem explorations and modeling techniques to extend students' mathematical understanding;
- Including in the lesson plans activities that show connections of the concepts to real world situations.

On the other hand, the Accomplished mathematics teacher exceeds expectations in the following areas:

- Use of basic thinking skills;
- Use of the language of mathematics;
- Use of appropriate objects or situations to develop students' mathematical understanding.



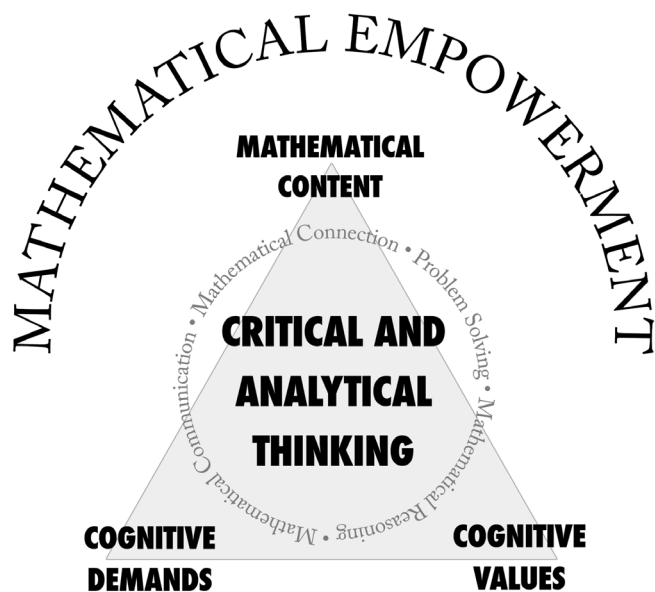
The Accomplished mathematics teacher greatly meets expectations indicated by the following behaviors:

- Passing performance evaluation;
- Reflecting in the lesson plan appropriate prerequisite knowledge;
- Arranging mathematical topics in their logical sequential order;
- Solving problems, analyzing errors, reasoning out, formulating decisions, making conjectures and proving theorems or conjectures;
- Use of varied ways of solving a problem, reasoning and making decisions;
- Generalizing and extending results of solving problems;
- Including in the lesson plans activities that show connections of the concepts to real world situations;
- Constructing connections within and among the branches of mathematics;
- Connecting mathematics to other disciplines.

The Accomplished mathematics teacher substantially meets expectations indicated by the following behaviors:

- Using varied ways of making conjectures and proving theorems or conjectures;
- Applying mathematical reasoning and appropriate technologies to develop concepts, procedures and conjectures;
- Generalizing reasoning skills and applying or extending them to other contexts;
- Using problem explorations and modeling techniques to extend students' mathematical understanding.

Both the Expert and Accomplished mathematics teachers are expected to substantially pass a well designed content examination for mathematics teachers, as are the Emerging and Novice teachers.



Mathematical Empowerment focusing on developing **Critical and Analytical Thinking** encompasses the following skills as well: *Problem Solving, Communicating Mathematically, Reasoning and Making Mathematical Connections*. The vision is to achieve the focus goal through the teaching of a solid mathematical content, the development of strong cognitive skills and the promotion of desirable cognitive values.

Figure 5. Mathematics Framework for Philippine Basic Education



CHAPTER 5

MATHEMATICAL PEDAGOGICAL KNOWLEDGE OF TEACHERS

An effective mathematics teacher is one that does not only know mathematics deeply but also knows a lot about the teaching of mathematics. Shulman's (1986) pioneering categorization of this type of knowledge among mathematics teachers which he calls Pedagogical Content Knowledge (PCK) sparked much interest in how one can develop such knowledge. In this framework, we refer to this type of knowledge as Mathematical Pedagogical Knowledge (MPK) to clearly indicate the type of knowledge necessary in carrying out the teaching of mathematics justified by specific principles.

Mathematical Pedagogical Knowledge includes four domains:

- School mathematics curriculum (Principle 7)
- Students' cognition of mathematics (Principle 2)
- Tasks of mathematics teaching (Principles 3 and 4)
- Mathematical discourse (Principles 3 and 4)

The principles alongside each domain are those that justify MPK as a very important knowledge domain for teachers.

Knowledge of School Mathematics Curriculum

Mathematics teachers understand instructional planning and design instruction based upon their knowledge of school mathematics curriculum, curricular frameworks and goals, students, learning theories and the community among others. They must know the contents of school mathematics curriculum and how

curriculum is developed and designed mainly for the purpose of knowing how to use it and plan for its effective implementation. Teachers must understand curriculum in ways that allow them to explain and “unpack” ideas to enable them to help students learn (Ball & Bass, 2004). Additionally, mathematics teachers should know the logical and developmental explanations for the sequencing of mathematical topics so that they know how to adjust and change lesson sequences when needed.

In order to have a good understanding of what goes into school mathematics curriculum, teachers must have a good knowledge of the wider field of mathematics, its contents, its research areas and recent developments and applications beyond school mathematics. A sufficient knowledge of these helps mathematics teachers understand the expectations at the level of school mathematics.



Knowledge of Students' Cognition of Mathematics

More than just knowing particular learners, mathematics teachers must have sufficient understanding of how students learn, in general, and understand mathematics. This will help the teachers know what mathematical tasks are appropriate and what teaching methodologies are effective in promoting students' development of critical and analytical thinking, problem solving and other mathematical skills (NRC, 2001).

Mathematics teachers need to know how to interpret students' mathematical thinking as indicated by their written works and verbal explanations, including the errors students make so that they could determine students' level of mathematical understanding.

Mathematics teachers must know the different mathematical tasks from which students learn mathematics best. They must know how to engage students in significant mathematical tasks so that they could raise students' level of understanding of mathematics and improve students' mathematical skills. This implies that teachers know what teaching approaches are useful in developing specific skills and knowledge in mathematics.

Knowledge of the Tasks of Mathematics Teaching

This domain includes one's knowledge of the different tasks related to the teaching of mathematics: problem selection, assessment, technology use and materials development. Mathematics teachers need to know how to select the best problems and exercises to give so that students learn the different methods and approaches to solving problems and the applications of concepts.

Mathematics teachers also have to know different ways of assessing students' understanding of mathematics and their mathematical skills. They know how to use these to support and encourage the continuous development of students. Likewise, teachers could use assessment results to further improve their teaching skills.

With the availability of technology, mathematics teachers must know when to use technology and how to use the different technological tools in teaching mathematics.

Last but not least, mathematics teachers should know how to continuously develop teaching aids and other materials specific to mathematics. While there may be available materials, teachers know their students and students' contexts best, hence, they are in the best position to develop teaching aids and materials that cater specifically to their students.

Knowledge of Mathematical Discourse

Discourse is important in mathematics. It is one way of engaging students deeply in mathematics. The value of mathematical discourse as a pedagogical tool cannot be overemphasized. It is through discourse that teachers are able to detect students' faulty understanding of mathematical concepts as well as appreciate students' deep and broad understanding of mathematical theories. Mathematical discourse includes simple verbal explanations and short discussions among students and between students and teacher. Mathematics teachers must know how to plan, implement and manage mathematical discourse in the classroom. Communication and verbal skills in mathematics are as important as all other mathematical skills.



Table 3. Performance Indicators for Mathematical Pedagogical Knowledge and Teacher Growth Levels
 N - Novice; Em - Emerging; A - Accomplished; Ex - Expert

- † - Barely meets the expectation * - Greatly meets the expectation
 ★ - Substantially meets the expectation ✱ - Exceeds expectation

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
School Mathematics Curriculum	<i>The teacher knows and understands the basic principles of planning and designing curricula.</i>				
	The teacher demonstrates understanding of the principles in the Mathematics Framework for Basic Education (MFBE) and incorporates these principles into the planning and design of instruction.	†	★	★	✱
	The teacher selects learning materials that are appropriate for the discipline and curriculum goals based on the principles established in the Mathematics Framework for Teacher Education (MFTE).	†	★	★	✱
	<i>The teacher knows the contents of school mathematics curricula.</i>				
	The teacher establishes learning expectations based on curriculum contents and learning competencies recommended by the MFBE.	†	★	★	✱
	<i>The teacher knows the content sequences of the mathematics curricula.</i>				
	The teacher recognizes and understands the prerequisite concepts for each and every content material in the curriculum.	†	★	★	✱
	The teacher adjusts the sequence of content material as needed without jeopardizing the cognitive requirements.	†	★	★	✱
	<i>The teacher understands the need to constantly develop curriculum materials.</i>				
The teacher creates teaching resources, mathematical tasks and curriculum materials based on the principles of the MFBE and MFTE.		†	★	★	
Students' Cognition of Mathematics	<i>The teacher knows and understands mathematical learning theories.</i>				
	The teacher applies results from recent studies on mathematical processes and learning theories to develop basic and higher order thinking skills among students.	†	★	★	✱
	<i>The teacher knows and understands students' mathematical thinking and beliefs.</i>				
The teacher uses students' prior knowledge and information on students' mathematical thinking and beliefs to build students' understanding of mathematical concepts.	†	★	★	✱	



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Students' Cognition of Mathematics (<i>continuation</i>)	The teacher analyzes and evaluates the mathematical thinking of students using varied methods of inquiry.	✦	★	★	✨
	The teacher recognizes and encourages students' invented strategies in solving problems (e.g., counting strategies, mental mathematics).	✦	★	★	✨
	The teacher probes students' mathematical thinking and tests their understanding according to the process of inquiry and standards offered by the MFBE.		✦	★	✨
	The teacher identifies and corrects students' misconceptions.		✦	★	✨
Tasks of Mathematics Teaching	<i>The teacher knows how to engage students in mathematical tasks.</i>				
	The teacher implements instructional activities that allow students to be active learners of mathematics.	✦	★	★	✨
	The mathematical tasks that the teacher poses provoke, engage and challenge students.		✦	★	★
	The teacher creates a learning environment where students formulate their own problems and solve problems in creative ways. In this environment, students are given opportunities to reflect upon the problems, their solutions (or alternative solutions), generalizations or extensions.		✦	★	★
	The teacher evaluates, selects, and uses appropriate learning activities that develop students' basic and higher order thinking skills.	✦	★	★	✨
	<i>The teacher knows and understands the general theories of assessment.</i>				
	The teacher uses formative assessments to gather cross-sectional views of students' work, progress and performance to adjust and improve instruction.	✦	★	★	✨
	The teacher utilizes appropriate and useful summative assessments to determine the impact of mathematical learning activities on students' learning.	✦	★	★	✨
	The teacher uses authentic assessments and constructs rubrics that reflect knowledge of students' thinking processes and errors.		✦	★	★

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Tasks of Mathematics Teaching (<i>continuation</i>)	<i>The teacher appropriately integrates technology into mathematical activities.</i>				
	The teacher creates a learning environment that allows students to use calculators, computers and other technological devices when needed to enhance mathematical understanding of concepts and processes.	★	★	★	★
	The teacher evaluates, selects and uses appropriate technologies as aid to effective mathematics teaching and learning.		★	★	★
Mathematical Discourse	<i>The teacher understands the need for mathematical discourse.</i>				
	The teacher plans and designs instruction to include discussions and genuine exchange of mathematical ideas between and among students and teacher.		★	★	★
	The teacher encourages students to make connections and develop a coherent framework for mathematical ideas.		★	★	★
	The teacher examines effects of tasks and mathematical discourse on students' mathematical knowledge, skills and dispositions.		★	★	★
	<i>The teacher knows how to implement and manage mathematical discourse.</i>				
	The teacher creates a learning environment that promotes critical thinking, reasoning and analytical thinking among students through discussions and mathematical discourse.		★	★	★
	The teacher encourages and accepts the use of the following to enhance intellectual discourse: <ul style="list-style-type: none"> • Computers, calculators and other technology; • Concrete materials as models; • Pictures, diagrams, tables and graphs; • Invented and conventional terms and symbols; • Metaphors, analogies and stories; • Written hypotheses, explanations and arguments. 		★	★	★
	The teacher encourages and facilitates students' sharing and recording of mathematical ideas in varied ways (words, symbols, diagrams, models, etc.) to develop reasoning skills.	★	★	★	✱
	<i>The teacher understands the language of mathematics.</i>				
	The teacher organizes and consolidates mathematical thinking through oral, written or symbolic communication and expresses this clearly and coherently and encourages students to do the same.	★	★	★	✱

Quest for Excellence in Mathematical Pedagogical Knowledge

It is helpful to pull together the ideas offered in Table 3 so that mathematics teachers and educators know how to achieve excellence in the teaching of mathematics content. The model for excellence is the Expert mathematics teacher who has a wider edge over the Accomplished teacher in this domain of knowledge. However, the Accomplished mathematics teacher is just as competent in some aspects of mathematics pedagogical knowledge and, therefore, must be emulated as much. In this section, the expectations of Expert and Accomplished mathematics teachers with respect to mathematics pedagogical knowledge are summarized.

Based on Table 3, Expert mathematics teachers exceed expectations indicated by the following behaviors:

- Demonstrating understanding of the principles in the MFBE and incorporates these principles into the planning and design of instruction;
- Selecting learning materials that are appropriate for the discipline and curriculum goals based on the principles established in the MFTE;
- Establishing learning expectations based on curriculum contents and learning competencies recommended by the MFBE
- Recognizing and understanding the prerequisite concepts for each and every content material in the curriculum;
- Adjusting the sequence of content material as needed without jeopardizing the cognitive requirements;
- Applying results from recent studies on mathematical processes and learning theories to develop basic and higher order thinking skills among students;
- Using students' prior knowledge and information on students' mathematical

- thinking and beliefs to build students' understanding of mathematical concepts;
- Analyzing and evaluating the mathematical thinking of students using varied methods of inquiry;
- Probing students' mathematical thinking and testing their understanding according to the process of inquiry and standards offered by the MFBE;
- Recognizing and encouraging students' invented strategies in solving problems (e.g., counting strategies, mental mathematics);
- Identifying and correcting students' misconceptions;
- Implementing class activities that allow students to be active learners of mathematics;
- Evaluating, selecting and using appropriate learning activities that develop students' basic and higher order thinking skills;
- Using formative assessments to gather cross-sectional views of students' work, progress and performance to adjust and improve instruction;
- Utilizing appropriate and useful summative assessments to determine the impact of mathematical learning activities on students' learning;
- Encouraging and facilitating students' sharing and recording of mathematical ideas in varied ways (words, symbols, diagrams, models, etc.) to develop reasoning skills;
- Organizing and consolidating mathematical thinking through oral, written or symbolic communication and expresses this clearly and coherently and encourages students to do the same.

The Expert mathematics teacher greatly meets expectations as indicated by the following behaviors:

- Creating teaching resources, mathematical tasks and curriculum materials based on the principles by the MFBE and MFTE;
- Developing mathematical tasks that provoke, engage and challenge students;



- Creating a learning environment where students formulate their own problems and solve problems in creative ways;
 - Using authentic assessments and constructing rubrics that reflect knowledge of students' thinking processes and errors;
 - Creating a learning environment that allows students to use calculators, computers and other technological devices when needed to enhance mathematical understanding of concepts and processes;
 - Evaluating, selecting and using appropriate technologies as aid to effective mathematics teaching and learning;
 - Planning and designing instruction to include discussions and genuine exchange of mathematical ideas between and among students and teacher;
 - Encouraging students to make connections and developing a coherent framework for mathematical ideas;
 - Examining effects of tasks and mathematical discourse on students' mathematical knowledge, skills and dispositions;
 - Creating a learning environment that promotes critical thinking, reasoning and analytical thinking among students through discussions and mathematical discourse;
 - Encouraging and accepting the use of different technologies, mathematical representations and arguments to enhance intellectual discourse.
- Notably, the Accomplished mathematics teacher greatly meets expectations in some of the following areas:
- Understanding of the basic principles of planning and designing curricula;
 - Understanding of the contents and content sequences of school mathematics curricula
 - Understanding of mathematical learning theories;
 - Understanding of students' mathematical thinking and beliefs;
 - Understanding of the general theories of assessment;
 - Developing active learning environments that allow for use of technologies and ensuring that these environments help develop both basic and higher order thinking skills;
 - Knowledge of how to implement and manage mathematical discourse by specifically
 - encouraging and facilitating students' sharing and recording of mathematical ideas in varied ways (words, symbols, diagrams, models, etc.) to develop reasoning skills;
 - organizing and consolidating mathematical thinking through oral, written or symbolic communication and expressing this clearly and coherently and encouraging students to do the same.

Mathematics Pedagogical Knowledge requires a lot of experience before a teacher can truly become an expert in it. Thus, it is in this area that Expert and Accomplished teachers have much greater advantage over Emerging and Novice mathematics teachers.

Philippine Teacher Education

Principle 2

Mathematics must be real to students and therefore, mathematics teachers should be mindful of students' contexts when teaching mathematics.



CHAPTER 6

GENERAL PEDAGOGICAL KNOWLEDGE AND MANAGEMENT SKILLS OF TEACHERS

The combination of a broad knowledge of mathematical pedagogies and general pedagogies makes a potent force that could ensure a high quality mathematics teaching in schools. Every teacher should have a good, solid general pedagogical knowledge. This knowledge includes the understanding of students and their development as learners, knowledge of general teaching methodologies and classroom processes, including lesson planning techniques, knowledge of theories of assessment and understanding of professional responsibilities. Principles 2, 3, 4, 5 and 7 support the significance of these knowledge domains for mathematics teachers.

Mathematics teachers are also expected to be armed with skills in managing several aspects of the teaching and learning processes – the classroom environment, learning resources and the students. This reflects the inherent role of teachers as facilitators of learning. To ensure that genuine learning occurs in the classroom, mathematics teachers must be able to handle everyday learning situations in the most efficient and democratic way possible.

Knowledge of Students, their Cognitive Development and Contexts

This domain includes teachers' knowledge of the general characteristics of students that are under their care – their developmental and cognitive background, aptitude, the different situations and contexts that they come from, their beliefs and attitudes as well as social and cultural backgrounds. Teachers need to know this in order to plan appropriate methodologies

for teaching not only mathematics but also little lessons in life and for developing them as learners who are the future leaders of the country.

Knowledge of General Teaching Techniques and Classroom Processes

Mathematics teachers need not only know specific teaching techniques for mathematics; a good solid knowledge of general teaching techniques that have been proven effective such as group work, practical work, investigative studies, class presentations, etc., can help expand the repertoire of techniques that are tailor-fitted to mathematics. Knowledge of classroom processes helps mathematics teachers keep a structure that assists students in developing good study habits. Techniques in checking attendance and homework, for example, are important to learn in order to maximize the potential of each without sacrificing time to



complete them. Other classroom processes that are important in any class include taking tests (oral and written), integrating library work, recitation, etc. Learning the techniques on how to conduct or implement these would make classroom processes more efficient, exciting and interesting to the learners.

Knowledge of Theories of Assessment

Although mathematics teachers are generally armed with techniques in assessing students' mathematical knowledge and understanding, mathematics teachers also ought to know the different theories of assessment. These theories explain why certain assessment formats, for example, would not work in certain situations but would work in some.

When knowledge is anchored on theory, knowledge lasts longer and is held more strongly. Thus, it is important for mathematics teachers to know and understand the different theories of assessing knowledge. These theories would make their understanding of assessment richer, enabling them to shift and adjust practices as called for by specific situations and contexts.

Knowledge of Professional Responsibilities

Teachers' professional responsibilities include knowing and understanding the national and local standards of education and communicating effectively with parents of pupils, among many others.

Mathematics teachers have the responsibility to know and understand the standards for maintaining good quality education in mathematics. These standards ensure that pupils do receive the kind of education that they deserve. These standards guide mathematics teachers in their implementation of the curriculum.

Mathematics teachers also need to develop their ability to communicate with parents effectively. Parents deserve to know an accurate and fair assessment of their children's progress in school mathematics. Who else would be in the best position to do this than mathematics teachers? They are the ones who teach and nurture students' knowledge and understanding of mathematics. They ought to know how to give useful feedback to parents regarding their children's performance in mathematical activities and more importantly, their understanding of mathematics.

Table 4 shows the performance indicators for general pedagogical knowledge and the levels at which these indicators are expected to be manifested.

Philippine Teacher Education

Principle 3

Mathematics is best learned when students are actively engaged.



Table 4. Performance Indicators for General Pedagogical Knowledge and Teachers' Growth Levels
N - Novice; Em - Emerging; A - Accomplished; Ex - Expert

- ✦ - Barely meets the expectation ✨ - Greatly meets the expectation
 ✧ - Substantially meets the expectation ✨ - Exceeds expectation

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Students	<i>The teacher knows the general background and contexts that the students come from.</i>				
	The teacher uses different learning theories and the principles of educational psychology to understand students, their social and cultural backgrounds, their individual differences and unique contexts so as to encourage autonomy, responsibility and respect for others.	✦	✧	✨	✨
	<i>The teacher is knowledgeable about the nature of cognitive development of students.</i>				
	The teacher uses appropriate motivational strategies (including use of visual aids and appropriate technology) to arouse and sustain the interests of students and build on and connect concepts.	✦	✧	✨	✨
	The teacher motivates students by integrating the historical development of mathematics, contribution of cultures, communities and real life situations in appropriate lessons to develop meaningful conceptual understanding and connections to students' lives.	✦	✧	✨	✨
	The teacher uses methods of inquiry that address the learning needs of students and facilitates their conceptual understanding of mathematics.	✦	✧	✨	✨
Teaching and Classroom Processes	<i>The teacher knows general teaching techniques that extend to particular techniques useful in teaching mathematics.</i>				
	The teacher displays knowledge of group work, practical work, investigative studies and class presentations as different ways of teaching mathematics to students.	✦	✧	✨	✨
	<i>The teacher knows basic classroom processes like checking of attendance, homework, etc.</i>				
	The teacher sets definite procedures for checking attendance and homework and is able to keep track of other procedures necessary to keep order in teaching.	✦	✧	✨	✨



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Theories of Assessment	<i>The teacher understands the psychological bases for assessment.</i>				
	The teacher uses assessment methods and techniques and makes them an integral part of instruction to provide information and guidance in making instructional decisions.	★	★	★	★
	<i>The teacher knows some general assessment techniques that are applicable to mathematics classes.</i>				
	The teacher writes useful and accurate assessments of students.	★	★	★	★
	The teacher appropriately uses a variety of assessment methods to evaluate students' understanding, progress and performance.		★	★	★
	The teacher uses assessment strategies to make students aware of their strengths and needs and encourage them to set their own personal goals for learning.		★	★	★
	<i>The teacher knows the appropriate uses of assessment.</i>				
	The teacher uses assessment results to diagnose student learning needs, align and modify instruction and design teaching strategies.	★	★	★	★
Professional Responsibilities	<i>The teacher is familiar with and understands the national and local standards in mathematics education.</i>				
	The teacher incorporates the MFBE principles and standards into the planning, designing and execution of instruction.	★	★	★	★
	<i>The teacher knows how to effectively communicate with parents, peers and supervisors.</i>				
	The teacher displays professionalism in meeting with parents, peers and supervisors.	★	★	★	★
	The teacher keeps neat and accurate records for reporting students' progress and achievements.	★	★	★	★
	The teacher renders good judgment in handling and discussing sensitive issues involving students.	★	★	★	★
	The teacher adheres to certain protocols and procedures of the institution when reporting students' progress and achievement.	★	★	★	★



Quest for Excellence in General Pedagogical Knowledge and Knowledge of Management Skills

Expertise in mathematics teaching includes excellence in knowledge of pedagogies that are not particular to mathematics but are necessary to ensure learning and to maintain an atmosphere of learning. More and more, it is important for mathematics teachers to know how to manage learning resources as well.

In the quest for excellence in this knowledge domain, it is useful to note that the Expert mathematics teacher exceeds expectations in the following areas:

- Use of different learning theories and the principles of educational psychology to understand students;
- Use of appropriate motivational strategies to arouse and sustain the interests of students and build on and connect concepts;
- Motivation of students by integrating the historical development of mathematics, contribution of cultures, communities and real life situations in appropriate lessons to develop meaningful conceptual understanding and connections to students' lives;
- Use of methods of inquiry that address the learning needs of students and facilitate their conceptual understanding of mathematics;
- Knowledge of group work, practical work, investigative studies and class presentations as different ways of teaching mathematics to students;
- Setting of definite procedures for checking attendance and homework and is able to keep track of other procedures necessary to keep order in teaching;
- Use of assessment methods and techniques as an integral part of instruction to provide information and guidance in making instructional decisions;

- Writing of useful and accurate assessments of students;
- Use of assessment results to diagnose student learning needs, align and modify instruction and design teaching strategies;
- Incorporation of the MFBE principles and standards into the planning, designing and execution of instruction;
- Display of professionalism when meeting with parents, peers and supervisors;
- Keeping of neat and accurate records for reporting students' progress and achievements;
- Rendering of good judgment in handling and discussing sensitive issues involving students;
- Adherence to certain protocols and procedures of the institution when reporting students' progress and achievement.

The Accomplished mathematics teacher greatly exceeds expectations in the same areas.

Knowledge of Management Skills

In order to maintain a suitable learning atmosphere in the classroom, mathematics teachers must have sufficient knowledge in managing the classroom, the available resources in the classroom and in school and students.

Table 5 shows the performance indicators for knowledge of management skills and the levels at which these indicators are expected to be manifested.



Table 5. Performance Indicators for Knowledge of Management Skills and Teachers' Growth Levels
N - Novice; Em - Emerging; A - Accomplished; Ex - Expert

- ✦ - Barely meets the expectation ✨ - Greatly meets the expectation
 ★ - Substantially meets the expectation ✨ - Exceeds expectation

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Classroom as a learning environment	<i>The teacher knows the effects of larger or smaller class sizes and multi-grade classes.</i>				
	The teacher can adjust teaching methodologies to capitalize on the strengths of large or small classes and of having multi-grade classes.		✦	★	✨
	<i>The teacher knows how to optimize the classroom space to enhance learning.</i>				
	The teacher can accommodate a variety of activities for students according to the amount of physical space available.		✦	★	✨
	The teacher provides a safe, comfortable and intellectually stimulating environment that is conducive for learning mathematics.		✦	★	✨
	<i>The teacher recognizes the role of errors in fostering better understanding of mathematics concepts.</i>				
	The teacher maintains a supportive atmosphere of trust and respect for students' ideas.		✦	★	✨
	The teacher provides the necessary scaffolding for students to correct their own errors.		✦	★	✨
	<i>The teacher understands the value of keeping a learning community in the classroom that adheres to democratic and collaborative principles.</i>				
	The teacher shows respect for each individual student and encourages everyone in the class to do the same. The teacher avoids acts that discriminate against specific groups of learners (e.g., slow learners).	✦	★	✨	✨
	The teacher maintains an atmosphere that allows students to work collaboratively with one another.	✦	★	✨	✨
	The teacher uses appropriate language, encourages genuine exchange of ideas in the form of discussions and promotes communication during class.	✦	★	✨	✨



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Classroom, school and community resources	<i>The teacher knows the available resources, facilities, equipment and other materials available for use.</i>				
	The teacher provides materials that are not available in school, whenever possible.	★	★	★	★
	The teacher displays resourcefulness and creativity in providing materials that are not readily available.	★	★	★	✱
	The teacher routinely checks all resources, facilities and equipment under one's care.	★	★	★	✱
	The teacher makes use of varied resources, facilities, equipment and other materials appropriate to the lessons.		★	★	★
	<i>The teacher recognizes the indigenous resources found in the community.</i>				
	The teacher promotes the use of indigenous resources through mathematical investigations, explorations and projects.	★	★	★	✱
	The teacher provides learning experiences that promote awareness and appreciation of mathematics that is rooted in the students' culture.		★	★	★
	<i>The teacher is aware of the human resources available in school and in the community.</i>				
	The teacher invites resource persons for learning activities when appropriate and possible.	★	★	★	✱
Students and Students' behavior	<i>The teacher understands the general psychological and cognitive development of students.</i>				
	The teacher displays sensitivity to, and draws on, students' diverse background experiences and dispositions.		★	★	★
	The teacher accommodates different learning abilities and diverse learning styles of students.		★	★	★
	The teacher motivates students to become productive and self-regulated learners.		★	★	★
	The teacher observes, listens and gathers useful information about students to assess what and how they are learning.		★	★	★
	<i>The teacher knows how to maintain proper behavior and discipline in the classroom.</i>				
	The teacher utilizes correct discipline principles in the classroom.	★	★	★	✱
	The teacher handles unanticipated events and critical incidents in the classroom wisely, efficiently and properly.		★	★	★

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Students and Students' behavior (<i>continuation</i>)	<i>The teacher knows motivational strategies to sustain students' interest.</i>				
	The teacher positively uses sense of humor, "light moments" or appropriate body language to enhance learners' enthusiasm.	★	★	★	★
	The teacher monitors students' participation in discussion and decides when and how to encourage each student to participate.		★	★	★
	The teacher encourages students to listen, respond and pose questions to the teacher and to one another.	★	★	★	★

Quest for Excellence in Knowledge of Management Skills

This is perhaps one domain in which even expert mathematics teachers are tremendously challenged. Mathematics teaching is a juggling act, requiring teachers to balance their attention, time, skills and expertise in different domains in order to create a wonderfully integrated learning environment for students.

In spite of some difficulties involved, however, the Expert mathematics teacher would still exceed expectations as indicated by the following behaviors:

- Shows respect for each individual student and encourages everyone in the class to do the same;
- Maintains an atmosphere that allows students to work collaboratively with one another;
- Uses appropriate language, encourages genuine exchange of ideas in the form of discussions and promotes communication during class;
- Provides materials that are not available in school, whenever possible;
- Displays resourcefulness and creativity in providing materials that are not readily available;
- Promotes the use of indigenous resources through mathematical investigations, explorations and projects;

- Invites resource persons for learning activities when appropriate and possible;
- Utilizes correct discipline principles in the classroom;
- Positively uses sense of humor, "light moments" or appropriate body language to enhance learners' enthusiasm;
- Encourages students to listen, respond and pose questions to the teacher and to one another.

On the other hand, the Expert mathematics teacher greatly exceeds expectations in the following areas:

- Adjustment of teaching methodologies to capitalize on the strengths of large or small classes and of having multi-grade classes;
- Accommodation of a variety of activities for students according to the amount of physical space available;
- Provision of a safe, comfortable and intellectually stimulating environment that is conducive for learning mathematics;
- Maintenance of a supportive atmosphere of trust and respect for students' ideas;
- Use of the necessary scaffolding techniques for students to correct their own errors;
- Use of varied resources, facilities, equipment and other materials apt for the lesson;
- Routine check-up of all resources, facilities and equipment under one's care;



- Provision of learning experiences that promote awareness and appreciation of mathematics that is rooted in the students' culture;
- Sensitivity to students in order to draw on their diverse background experiences and dispositions;
- Accommodation of different learning abilities and diverse learning styles of students;
- Motivation of students to become productive and self-regulated learners;
- Gathering of useful information about students to assess what and how they are learning;
- Wise, efficient and proper handling of unanticipated events and critical incidents in the classroom;
- Monitoring of students' participation in discussion in order to decide when and how to encourage each student to participate.

Philippine Teacher Education

Principle 4

Mathematics can never be learned in an instant, but rather requires lots of work and the right attitude.

Principle 5

All students regardless of sex, culture, socio-economic status, religion and educational backgrounds have the right to learn and be taught good and correct mathematics.



CHAPTER 7

MATHEMATICAL DISPOSITION AND PROFESSIONAL DEVELOPMENT OF TEACHERS

The last but not the least component of competencies expected of mathematics teachers considers the affective aspects, or the so-called person in a mathematics teacher. Before subject matter and teaching proficiency, mathematics teachers must first be models of good mathematical disposition. As defined, mathematical disposition is the teachers' ability to make decisions for themselves that would impact on students' learning and on their own professional development. It includes mathematics teachers' beliefs and attitudes, their inclination to use mathematics and their willingness to reach out to others. Such disposition inevitably results in a transfer of the same disposition to students. This, of course, is most desired.

Teachers' mathematical disposition can be thought of according to the natural consequence and objects of such disposition. Their mathematical disposition could influence students' development of positive attitudes and motivation to learn mathematics. Likewise, mathematical disposition could affect teachers' self-efficacy and desire for professionalization. Thus, teachers' mathematical disposition could be directed towards self, students or others.

Studies have shown that mathematics teachers' beliefs and attitudes, which are manifested in their mathematical disposition and are reflected in their teaching, influence and affect their students' own beliefs and attitudes. Good mathematical disposition among teachers lead to increased positive attitudes among learners, which naturally result in increased motivation to learn more mathematics.

When mathematics teachers display good mathematical disposition, there is also an inward direction towards self, which results

primarily in increased self-efficacy. This, in turn, leads to increased desire for further professional growth and development. Good mathematical disposition may also be directed to others, which includes fellow teachers and the community. This results in increased professional relationship and cooperation between the mathematics teacher and others. Mathematics teachers cannot function alone while trying to raise the quality of mathematics teaching and learning. Thus, they need to develop their collegial skills, enabling them to establish professional networks and partnerships through which a genuine exchange of ideas and sharing of skills could develop. This is important for mathematics teachers to grow professionally. Principles 7 and 9 justify the significance of this knowledge domain for teachers.

Table 6 indicates the performance indicators for good mathematical disposition and the levels of teachers' growth and development which these indicators are expected to be manifested.



Table 6. Performance indicators for enhanced mathematical disposition and professional development and teacher growth levels.

N - Novice; *Em* – Emerging; *A* – Accomplished; *Ex* – Expert

- ✦ - Barely meets the expectation ✨ - Greatly meets the expectation
 ★ - Substantially meets the expectation ✨ - Exceeds expectation

Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Self	<i>The teacher exhibits a disposition to engage in mathematical activities and to teach mathematics.</i>				
	The teacher exudes or communicates in an intrinsic way an appreciation for mathematics and a spirit for doing mathematics.	✦	★	★	✨
	The teacher uses mathematics and mathematical language to describe or explore real world phenomena.	✦	★	★	✨
	The teacher exhibits a healthy respect for systematic thinking, e.g., organize written arguments or solutions in a neat and logical order.	✦	★	★	✨
	The teacher exhibits self-confidence (as opposed to braggadocio or bluffing), flexibility (as opposed to dogmatism or one-track-mindedness) and curiosity (as opposed to complacency).		✦	★	✨
Student	<i>The teacher knows techniques that will foster students' mathematical disposition; that is, encourage students' confidence in doing mathematics.</i>				
	The teacher uses verbal cues during instruction that maximize learning among students.	✦	★	★	★
	The teacher turns students' mistakes into learning experiences and helps students realize that one can learn from mistakes; they are a part of the learning process.		✦	★	★
	The teacher engages students in mathematical discourse (as opposed to always lecturing).		✦	★	★
Others	<i>The teacher knows the requisites for professional growth.</i>				
	The teacher engages in analyses of one's own teaching by self, with peers, with superiors.		✦	★	★
	The teacher undertakes further study through graduate courses, in-service courses, seminars and other fora.		✦	★	★
	The teacher gets involved in research to understand students, mathematics, learning and other related elements in mathematics education.			★	★



Domain	Knowledge Description and Performance Indicator	N	Em	A	Ex
Others (<i>continuation</i>)	<i>The teacher understands the need to establish collegial partnerships and networks.</i>				
	The teacher actively participates in discussions and meetings among colleagues and fellow teachers.			★	✱
	The teacher gets involved in the production of learning support materials.			★	★
	The teacher volunteers to organize or reach out to potential partners and colleagues in different activities that enhance mathematics teaching.			★	✱
	The teacher advocates continuous professional development of teachers and actively engages in activities that promote the professional development of teachers.			★	✱

Quest for Excellent Mathematical Disposition and Professional Development for Teachers

This is one domain in which the Novice teacher is truly a neophyte in all sense of the word. Perhaps, this is due to the many demands of mathematics teaching that prevent a novice teacher from focusing too much on the self and development of the self. This document argues that the teacher's self is as important as the subject one is teaching as well as the students that one faces everyday.

To summarize, the Expert mathematics teacher does attend to the needs of the self and maintains a pleasant and enviable disposition truly worth emulating. The Expert mathematics teacher exceeds expectations in the areas indicated by the following behaviors:

- Exudes or communicates in an intrinsic way an appreciation for mathematics and a spirit for doing mathematics;
- Uses mathematics and mathematical language to describe or explore real world phenomena;
- Exhibits a healthy respect for systematic thinking, e.g., organize written arguments or solutions in a neat and logical order;

- Exhibits self-confidence (as opposed to bluffing), flexibility (as opposed to dogmatism or one-track-mindedness) and curiosity (as opposed to complacency);
- Actively participates in dialogues and meetings among colleagues and fellow teachers;
- Volunteers to organize or reach out to potential partners in varying activities that enhance mathematics teaching;
- Advocates continuous professional development of teachers and actively engages in activities that promote the professional development of teachers.

On the other hand, the Expert mathematics teacher greatly meets expectations in the areas indicated by the following behaviors:

- Uses verbal cues during instruction that maximize learning among students;
- Turns students' mistakes into learning experiences and helps students realize that one can learn from mistakes;
- Engages students in mathematical discourse;
- Engages in analyses of one's own teaching by self, with peers, with superiors;
- Undertakes further study through graduate courses, in-service courses, seminars and other fora;



- Gets involved in research to understand students, mathematics, learning, and other related elements in mathematics education;
- Gets involved in the production of materials that support learning.

The Accomplished mathematics teacher greatly meets expectations in the following areas indicated by the following specific behaviors:

- Exudes or communicates in an intrinsic way an appreciation for mathematics and a spirit for doing mathematics;
- Uses mathematics and mathematical language to describe or explore real world phenomena;
- Exhibits a healthy respect for systematic thinking;
- Uses verbal cues during instruction that maximize learning among students;

In the area of professional growth, the Accomplished mathematics teacher is expected to engage in activities that promote one's own development. It is the Expert teacher that exceeds expectations in this area. As an expert, the Expert mathematics teacher is a leader and a champion of the noble goals of excellent mathematics teaching.

Philippine Teacher Education

Principle 6

Assessment must be an integral part of mathematics instruction.

Principle 7

Mathematics as a field continues to develop and evolve. Therefore, the teaching of it must keep up with developments in the field.

Principle 8.

Technology plays an important role in the teaching and learning of mathematics. Mathematics teachers must learn to use and manage technological tools and resources well.



CHAPTER 8

AD INFINITUM: THE ROAD TO PROFESSIONALIZING MATHEMATICS TEACHERS

The road to excellence in mathematics teaching is difficult. Yet, there is a prevailing belief that mathematics teaching or teaching in general does not require an educational preparation that is as demanding and tough as that for scientists, engineers, doctors and lawyers. The knowledge domains and expectations laid out in the earlier chapters explicitly show that educating mathematics teachers is as difficult as the aforementioned professionals.

Educating the Future Mathematics Teacher

Mathematics teaching is a complex activity. The amount of knowledge required to be a competent and effective teacher is not found solely in the university classroom nor obtained from classroom teaching experience alone. Classroom knowledge and experience are necessary ingredients to develop competence in this field. Nevertheless, it is important to discuss other aspects of a desired preparatory educational program for future mathematics teachers. These are an excellent curriculum, competent professors, a well-planned practicum program and a credible professional accreditation system.

Excellent Curriculum

An excellent curricular program for future mathematics teachers is one that adheres to the vision of this framework. Definitely, it requires an excellent combination of courses that teach mathematics content, mathematics pedagogy, general pedagogy and research. Like any university program, a curricular program for

future mathematics teachers must conform to the highest standards in the field. It should not scrimp on opportunities for future mathematics teachers to broaden and deepen their knowledge of mathematics, techniques of teaching and useful research about the teaching and learning of mathematics.

Competent Teacher Educators

The quality of a preparatory program for mathematics teachers is affected by the professors and staff who teach and organize instruction and assessments in the program. Competent teacher educators are a necessity if we are to ensure high quality preparation for future mathematics teachers. Substandard teaching of mathematics is not acceptable. Only the most competent professors must teach mathematics to future mathematics teachers so that in addition to the content, these teachers would be able to acquire the discipline, rigor and habits needed to successfully learn the subject. Likewise, the teaching of pedagogies that limit mathematics learning to a mere textbook-blackboard-paper



experience should be minimized. What is needed from mathematics education professors is the teaching of a dynamic set of techniques that allow for more exciting and effective ways of learning mathematics.

The bottom line is that future mathematics teachers need good role models and it is the professors who could provide the knowledge and guidance needed as teachers go through their four years of education in university.

Well-planned Practicum Program

A well-planned practicum program is essential in preparing future mathematics teachers. This program must be focused, rich and dynamic. Future teachers should be offered real opportunities to get immersed in the difficulties of an actual teaching job. They also need to be guided by expert teachers – teachers who would serve as additional role models – in the cooperating schools. The practicum program must include opportunities for reflection and processing of teaching experiences to allow for adjustments and modifications in their knowledge and understanding of the teaching dynamics. Program advisers who are accountable for the successful implementation of practicum programs have to ensure that these programs do provide the vital link between theory and practice of teaching.

Credible Professional Accreditation System

The first publicly acknowledged and legal step to being considered a professional is to pass a credible professional accreditation, which is often given in the form of a written examination. By law, all teachers must pass the LET offered by the Philippine Regulatory Commission (PRC). It is recommended, however, that other types of accreditation be allowed, as long as these meet rigorous professional standards. Professional teacher organizations could assist in developing a comprehensive and credible professional accreditation system for future

teachers that could possibly include practical and oral examinations. The government should impose stricter standards in the preparation and administration of examinations to make the accreditation system much more credible than it is now.

Continuing Education of Mathematics Teachers

The goal of any continuing professional development program for mathematics teachers should include facilitating their growth and movement toward the “Accomplished” and “Expert” levels that have been described in the earlier chapters. Whether these continuing programs are in the form of full-degree, post-baccalaureate programs or short courses, each professional development program for mathematics teachers must specify the targets that will clearly pave the way for growth of teachers towards the aforementioned levels.

Continuing professional development programs could be more systematically planned by classifying these according to three domains: nature of the program, type of education and duration.

By Nature

Continuing professional development programs that are designed to level off teachers’ competencies against the required basic competencies are referred to as Levelling-Off Programs. These programs are aimed at teachers who are considered to be at the pre-novice stage. The programs facilitate the movement of these teachers towards the novice level. They are designed to help teachers who do not meet the minimum standards or requirements for mathematics teaching develop basic knowledge and skills in mathematics and mathematics teaching. The nationwide program Project Rescue Initiatives in Science Education (RISE) is an example of such type of a program. Programs that are designed to provide



teachers with new competencies to broaden their knowledge within the same level of accomplishment are referred to as Updating Programs. These are programs that do not intend to raise the level of competency but to expand their knowledge, understanding and skills to include wider variety of contexts and situations on which the knowledge is applicable or useful.

Specializing Programs are programs that are designed to provide teachers with higher level of skills, competencies or knowledge so that they deepen their understanding of the subject matter. The intent is to upgrade or raise the quality of skills and understanding of mathematics and mathematics teaching. Post-baccalaureate programs such as the M.A., M.S. or Ph.D. programs are examples of these programs and so are the short-term, specialized courses that do not necessarily lead to a degree.

Programs that are designed to enhance mathematics teachers' skills in action-based classroom research are referred to as Professionalizing Programs. These are programs that are designed to help teachers develop themselves as a teaching colleague in a community of academic professionals. Examples of these programs are research internships or workshops facilitated by experts or sometimes, supervisors of schools.

By Type of Education

Continuing education professional development programs may be formal, informal or non-formal. Formal programs are school based and follow a fixed structure. Master's and doctoral programs are examples of this type. Informal programs are programs that are not school-based. Examples of this include training programs for writing action research and for writing research proposals for funding that are conducted by independent (non-school based) institutes or centers. The Knowledge Channel

classes are also examples of informal programs. Non-formal programs are programs that are school-based but not structured. Examples of these would include division-wide training programs in the use of computers for data manipulation and word processing.

By Duration

It is useful to describe continuing professional education programs according to their duration or length. Short-term continuous programs are those held continuously and regularly for up to 12 months. Long-term continuous programs are those held continuously and regularly but last for more than 12 months. Serial programs are those held regularly but not continuously and are offered in a series over a defined period of time.

Table 7 shows the nature of programs that cater best to enhancing particular teacher competencies, the perceived best format and duration of these programs. This table may be used as a guide in plotting the strengths and usefulness of programs that mathematics teachers find themselves enrolling in. It offers TEIs and all other teacher education providers a matrix on which they could plot the programs that they offer, enabling them to plan their courses better and with more focus. It is in the interest of mathematics teachers that the different education providers take the matrix seriously to avoid unproductive and repetitive programs that are offered aimlessly to mathematics teachers.



Table 7. Which Type of Program Enhances Teachers' Competencies?

Nature	Target Teacher Knowledge Domain	Type of Education	Duration
Levelling-Off	<ul style="list-style-type: none"> Mathematical Content Knowledge Mathematical Pedagogical Knowledge 	Formal	Long Term Serial
Updating	<ul style="list-style-type: none"> Mathematical Content Knowledge Mathematical Pedagogical Knowledge General Pedagogical Knowledge and Management Skills 	Formal Informal Non-formal	Short term Long term Serial
Specializing	<ul style="list-style-type: none"> Mathematical Content Knowledge Mathematical Pedagogical Knowledge 	Formal	Long Term
Professionalizing	<ul style="list-style-type: none"> Mathematical Disposition and Professional Development 	Non-formal Informal Formal	Short Term Long Term Serial

What Lies Ahead

Excellence in teaching must be a lifetime goal of all Filipino teachers. To achieve this requires quality education, mentorship by good role models and sincere earnestness on the part of the future teacher to succeed. The aim for excellence is synonymous to a desire for our students to succeed in their education. It is in the best interest of the future generations that mathematics teaching reach its highest standards.

In the hope of improving mathematics teaching in Philippine schools, this framework advocates continued efforts in the form of continuing programs for mathematics teachers' professional development that allow mathematics teachers to grow from being novices to becoming emerging and accomplished teachers. This must be the aim of all teacher education programs in mathematics.

Implications of what have been outlined in the preceding chapters include the need to rethink our policies and procedures for hiring mathematics teachers. Certainly, the competencies that have been outlined must be considered even for promotion policies and procedures as well.

The next step is to engage interested groups, academics and government in discussions on how this framework may best be utilized to improve the quality of the learning and teaching of Filipino mathematics teachers.

**Philippine Teacher Education
Principle 9**

Mathematics teachers must never stop learning.



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ACKNOWLEDGEMENTS

MATHTED and SEI wish to thank the following individuals, institutions and groups for responding to our calls for feedback, for serving as reviewers of the manuscript drafts and for participating in the various fora and workshops held from the years 2006 to 2007. Their invaluable contributions and insights were most important in the revision of the framework working draft.

Cooperating Institutions and Groups

DOST-Science Education Institute
 Ateneo de Manila University
 Commission on Higher Education Technical
 Panel for Teacher Education
 De La Salle University
 Department of Education
 Miriam College
 National Academy of Science and Technology
 Professional Regulation Commission
 State Universities and Colleges Teacher Educa-
 tion Association
 U.P. National Institute of Mathematics and Sci-
 ence Education Development

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